Vibration behaviour of cantilevered balcony slabs connected with a cantilever slab connection

Recommendation

For aesthetic reasons, balconies should be planned and constructed as slim and column-free as possible, which is contrary to the high static requirements and inevitably results in susceptibility to vibration. Human sensitivity to mechanical vibrations is subjective and the effect is frequency-dependent.

The human perception of mechanical vibrations depends on a number of factors not covered by the mechanical vibration system. There are no normative regulations or recommendations in technical literature [3] for the vibration limitation of cantilevered balcony slabs connected with a cantilever slab connection. For this reason, extensive calculations for the assumption of "slow walking" and "slow jumping" were carried out based on the data in literature [1].

Based on these results, the recommendation is to limit the initial resonance frequency in the system to ≥ 7.5 Hz. This recommendation is conservative because, among other things, damping was not considered.

This recommendation is product dependent, is based on the tests carried out and therefore applies exclusively to cantilever connections of the following manufacturers:

ECS

Engineered Construction Systems

1 Introduction

In modern building construction, balconies are an essential and standard component. As they are classed as additional living and usable spaces, balconies today should be as long, and cantilever as far as possible. At the same time, for aesthetic reasons, it is expected that the balconies are planned and constructed to be as slender and column-free as possible; this is contrary to the high static requirements and inevitably leads to susceptibility to vibrations. Architectural detailing and building physics requirements are additional complicating factors – from a vibration point of view – limiting the surface for connecting the balcony to the building.

Nowadays, prefabricated cantilever connectors are in common use. These connections consist of tension, compression and shear elements and thermal insulation. Compared to the reinforced concrete slab of the balcony and the reinforced concrete slab of the building, cantilever slab connections are much less rigid. If the maximum cantilever lengths recommended according to the slenderness conditions of EC2 (e.g. 2.0 – 2.2 m for a slab thickness of 20 cm) are observed, the vibration risk is low. Larger cantilevers can, however, influence the vibration behaviour and therefore affect the serviceability (cf. Ziegler [9]). When planning and dimensioning, in particular of larger cantilever balconies and the associated cantilever slab connections, it is therefore not only necessary to check and comply with the load safety and deformation requirements, but also to verify to what extent person-induced vibrations affect serviceability. As the currently valid and applicable technical regulations specify only a few of the requirements on the issues mentioned above, there was a need for more detailed study of the vibration behaviour of cantilevered balcony slabs using cantilever slab connection elements.

These evaluation and calculations include the verifications mentioned above on a simplified analytical model. The vibration behaviour of a cantilever beam restrained by a torsion spring with uniformly distributed mass was investigated. The system was loaded with various load scenarios typical for a balcony (pulsed and periodic), such as those caused by walking or jumping. Both the cantilever length and the thickness of the balcony slab as well as the characteristic working lines of the springs were varied. The maximum displacements occurring at the end of the cantilever arm and the maximum vibration velocities and accelerations at the position of the force were evaluated and presented graphically.

The mathematical and physical backgrounds are explained in Section 2. The results of the calculations are summarised in Section 3.

2 Analytical model and solution of the differential equation

The analytical model according to Ziegler [9] consists of a cantilevered element restrained by a torsion spring as shown in Fig. 1.

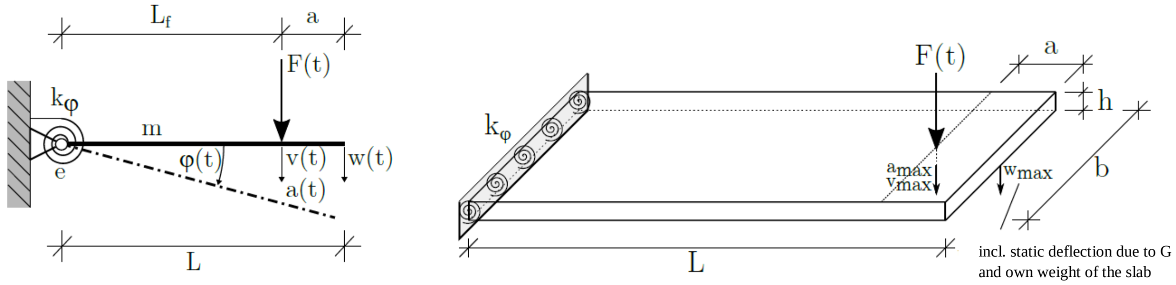

Fig. 1: Analytical model with all system parameters

Displays an oscillator with one degree of freedom which performs a rotational oscillation around the point of restraint (point "e" according to Fig. 1). The corresponding equation of motion is:

$$\ddot{\varphi}(t) + \frac{c_\varphi}{I_\Theta} \cdot \dot{\varphi}(t) + \frac{k_\varphi}{I_\Theta} \cdot \varphi(t) = \frac{F(t) \cdot \mathrm{L}_f}{I_\Theta} \tag{1}$$

This results in the resonance frequency:

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \cdot \sqrt{\frac{k_\varphi}{I_\Theta}} = \frac{1}{2\pi} \cdot \sqrt{\frac{k_\varphi}{\frac{M \cdot \mathrm{L}^2}{3}}} = \frac{1}{2\pi} \cdot \sqrt{\frac{3 \cdot k_\varphi}{L^3 \cdot A \cdot \rho}} \tag{2}$$

The system parameters are summarized in Table 1.

For the loading F(t), a periodic series development according to Bachmann et al. [3] is used, which is generally defined as follows where G = 0.8 kN:

$$F(t) = G + \sum_i G \cdot \alpha_i \cdot \sin\left(2\pi i f_p \cdot t - \phi_i\right) \tag{3}$$

In the present calculations, the loads "slow walking" and "slow jumping" are assumed by using the following values in Equation (3).

- slow walking: $\alpha_1 = 0.4;\ \alpha_2 = \alpha_3 = 0.1;\ \phi_2 = \phi_3 = \pi/2;\ f_p = 2\mathrm{Hz}$
- slow jumping: $\alpha_1 = 1.8;\ \alpha_2 = 1.3;\ \alpha_3 = 0.7;\ \phi_2 = \phi_3 = \pi\left(1 - f_p t_p\right);\ f_p = 2\mathrm{Hz}$

Designation	Description
L	Total length of the element
A	Cross-section area of the element
ρ	Density of the material
$M = L \cdot A \cdot \rho$	Mass of the element
k_φ	Torsional spring stiffness
$I_\Theta = \frac{M \cdot L^2}{3}$	Mass moment of inertia of the element with respect to the point of restraint
$\delta = \frac{c_\varphi}{2 \cdot I_\Theta}$	Damping constant
$D = \frac{\delta}{\omega}$	Degree of damping (damping factor acc. to Lehr)

Table 1: Summary of system parameters

The time-dependent rotation of the rod is obtained by inserting (3) into (1) and then solving the differential equation:

$$\varphi(t) = \left(A \cdot \sin(\varpi t) + B \cdot \cos(\varpi t)\right) \cdot e^{-\delta t} + \frac{G \cdot L_F}{k_\varphi} + \sum_{n=1}^{\infty} \frac{G \cdot L_F \cdot \alpha_i}{k_\varphi} \cdot \frac{1}{\left(1-\beta_n^2\right) + \left(2D\beta_n\right)^2} \cdot \left(\left(1-\beta_n^2\right) \cdot \sin\left(\Omega_n t - \phi_i\right) - 2D\beta_n \cdot \cos\left(\Omega_n t - \phi_i\right)\right) \tag{4}$$

Here Ω_n is the angular frequency of the excitation, $\beta_n = n \cdot \Omega_n / \omega$ is the ratio of the n[th] excitation circular frequency to the natural angular frequency of the oscillator and $\varpi = \omega \cdot \sqrt{1-D^2}$ is the natural angular frequency of the damped oscillation. The angular velocity and the angular acceleration result in the following:

$$\dot{\varphi}(t) = \varpi \cdot \left(A \cdot \cos(\varpi t) - B \cdot \sin(\varpi t)\right) \cdot e^{-\delta t} - \delta \cdot \left(A \cdot \sin(\varpi t) + B \cdot \cos(\varpi t)\right) \cdot e^{-\delta t} + \frac{G \cdot L_F}{k_\varphi} + \sum_{n=1}^{\infty} \frac{G \cdot L_F \cdot \alpha_i}{k_\varphi} \cdot \frac{\Omega_n}{\left(1-\beta_n^2\right)^2 + \left(2D\beta_n\right)^2} \cdot \left(\left(1-\beta_n^2\right) \cdot \cos\left(\Omega_n t - \phi_i\right) + 2D\beta_n \cdot \sin\left(\Omega_n t - \phi_i\right)\right) \tag{5}$$

and

$$\ddot{\varphi}(t) = \left(-\varpi^2 - \delta^2\right)\cdot\left(A\cdot\sin\left(\varpi t\right) + B\cdot\cos\left(\varpi t\right)\right)\cdot e^{-\delta t} - 2\delta\varpi\cdot\left(A\cdot\cos\left(\varpi t\right) - B\cdot\sin\left(\varpi t\right)\right)\cdot e^{-\delta t} + \frac{G\cdot L_F}{k_\varphi} + \sum_{n=1}^{\infty}\frac{G\cdot L_F\cdot\alpha_i}{k_\varphi} - 2\delta\varpi\cdot\left(A\cdot\cos\left(\varpi t\right) - B\cdot\sin\left(\varpi t\right)\right)\cdot e^{-\delta t} + \frac{G\cdot L_F}{k_\varphi} + \sum_{n=1}^{\infty}\frac{G\cdot L_F\cdot\alpha_i}{k_\varphi} \tag{6}$$

Assuming the initial rotation and the initial velocity are zero, the coefficients B and A are calculated as follows:

$$B = -\frac{G\cdot L_F}{k_\varphi} - \sum_{i=1}^{n}\frac{G\cdot L_F\cdot\alpha_i}{k_\varphi}\cdot\frac{1}{\left(1-\beta_n^2\right)^2 + \left(2D\beta_n\right)^2}\cdot\left(\left(1-\beta_n^2\right)\cdot\sin\left(-\phi_i\right) - 2D\beta_n\cdot\cos\left(-\phi_i\right)\right) \tag{7}$$

and

$$A = -\frac{\delta\cdot B}{\varpi} - \sum_{i=1}^{n}\frac{G\cdot L_F\cdot\alpha_i}{k_\varphi\cdot\varpi}\cdot\frac{\Omega_n}{\left(1-\beta_n^2\right)^2 + \left(2D\beta_n\right)^2}\cdot\left(\left(1-\beta_n^2\right)\cdot\cos\left(-\phi_i\right) + 2D\beta_n\cdot\sin\left(-\phi_i\right)\right) \tag{8}$$

The time curves of the displacement $\omega(t)$, the vibration velocity $\upsilon(t)$ and the vibration acceleration $a(t)$ resulting from Equations (4), (5) and (6) according to Fig. 1 can be calculated using a small angle approximation by multiplication with the total length L (for $\omega(t)$) and the lever arm of the load L_F (for $\upsilon(t)$ and $a(t)$). The maxima $\omega_{\max} = \max\{\omega(t)\}$, $\upsilon_{\max} = \max\{\upsilon(t)\}$ and $a_{\max} = \max\{a(t)\}$ occurring during the course of time are determined numerically.

3 Results

In this Section, exemplary results of the calculations for the analytical model according to Section 2 are summarized and presented graphically. Following the previous explanations, the calculation is carried out for the aforementioned loads "slow walking" and "slow jumping".

The calculations include the following parameters:

- Cantilever length $1.5\,\mathrm{m} \le L \le 3.5\,\mathrm{m}$
- Width of the slab cross section $b = 3.0\,\mathrm{m}$
- Distance of the load from the free edge $a = 0.4\,\mathrm{m}$
- Slab thicknesses $h = 0.16/0.20/0.25\,\mathrm{m}$
- Different, product-dependent torsional spring stiffnesses depending on the systems investigated

In principle, the calculation is carried out without damping. Recommendations for a frequency evaluation of the vibration accelerations are given in VDI 2057 [8] with reference to ISO 2631-1 [5]. In DIN 4150-2 [4] a frequency weighting of the vibration velocity is recommended. If both regulations are taken into account, the acceleration is decisive up to approx. 10 Hz, above this the velocity is decisive. Real limit values do not exist, however, as human perception of mechanical vibrations is subjective and the effect on humans is frequency-dependent.

Bachmann et al. [3] give an orientation on human perception of vibration effects (see Table 2). The transition from the acceleration limit to the speed limit at 10 Hz is also included here.

Description	**Frequency range 1 … 10 Hz Peak acceleration [mm/s²]**	**Frequency range 10 … 100 Hz Peak velocity [mm/s]**
just perceptible	34	0.5
clearly perceptible	100	1.3
disturbing/unpleasant	550	6.8
intolerable	1800	13.8

Table 2: Threshold values for maximum occurring vibration velocities and accelerations, according to [3].

The limit for intolerable perception was chosen for the investigations carried out. Fig. 2 shows the principle run of the result curve. Above 10 Hz the speed is decisive. The slight exceedances are considered acceptable. Below 10 Hz the acceleration is decisive, whereby the values increase with increasing cantilever length. For slow jumping, the limit value is exceeded at approx. 7.0 – 7.5 Hz. For slow walking, the limiting frequency is approx. 6.0 – 6.5 Hz. These patterns and limiting frequencies were observed in all investigations.

To examine the influence of damping, a conservative value of 2% was applied. In principle, the results do not differ from the results without damping. The limiting frequencies mentioned above are reduced by approx. 1 Hz. The results are shown in Fig. 3.

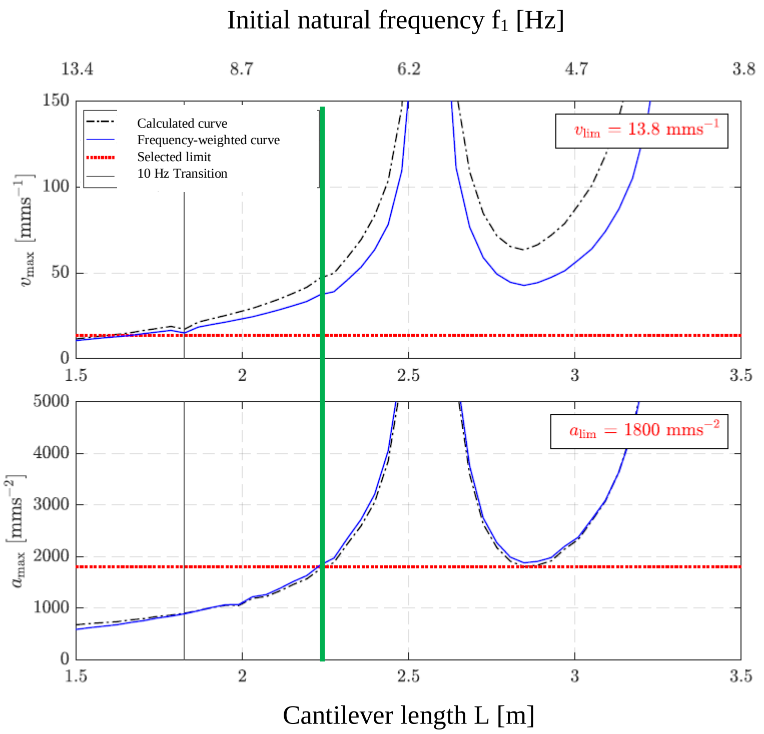

Fig. 2: Basic results for low torsion spring stiffness, h = 0.25 m, damping D = 0, slow jumping

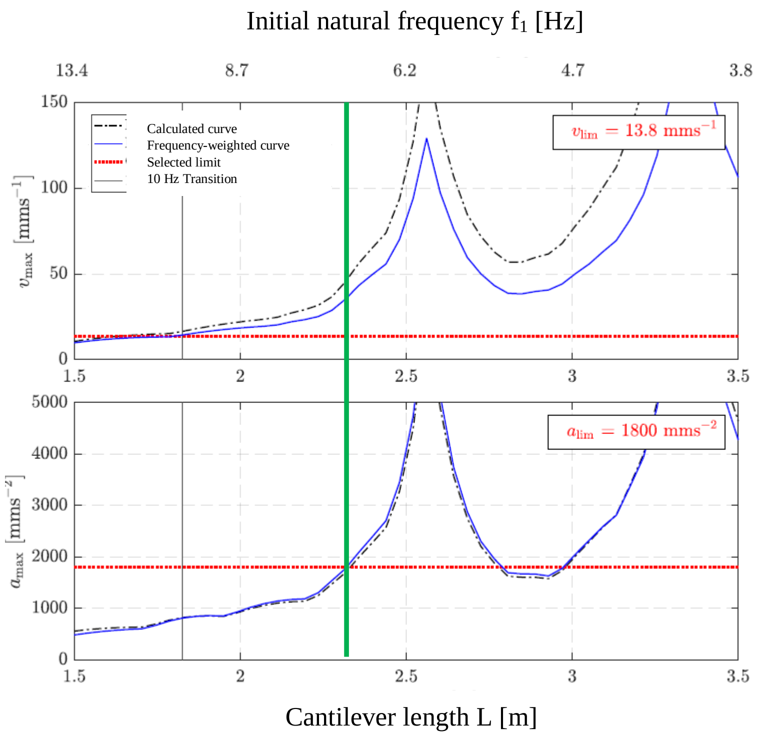

Fig. 3: Basic results for low torsion spring stiffness, h = 0.25 m, damping D = 2%, slow jumping

Based on these results, the recommendation is to limit the initial natural frequency in the system to ≥ 7.5 Hz. This recommendation is conservative because, among other things, damping was not considered.

4 Literature

[1] TU München, Lehrstuhl für Massivbau: Schwingungsverhalten auskragender und mit einem Kragplattenanschluss verbundener Balkonplatten. Kurzbericht und Darstellung der Ergebnisse. (Vibration behaviour of cantilevered balcony slabs connected with a cantilever slab connection. Summary report and presentation of results). Okt. 2018, nicht veröffentlicht/not published.

[2] Bachmann, H. und W. Ammann (1987): Vibrations in structures: induced by man and machines. Structural engineering documents. International Association for Bridge and Structural Engineering.

[3] Bachmann, H.; W. J. Ammann; F. Deischl; J. Eisenmann; I. Floegl; G. H. Hirsch; G. K. Klein; G. J. Lande; O. Mahrenholtz; H. G. Natke; H. Nussbaumer; A. J. Pretlove; J. H. Rainer; E.-U. Saemann und L. Steinbeisser (1995): Vibration Problems in Structures. Birkhäuser Basel. doi:10.1007/978-3-0348-9231-5.

[4] DIN 4150-2:1999-06 (1999): Erschütterungen im Bauwesen - Teil 2: Einwirkungen auf Menschen in Gebäuden (Vibrations in buildings - Part 2: Effects on persons in buildings).

[5] ISO 2631-1:1997-05 (1997): Mechanische Schwingungen und Stöße - Bewertung der Einwirkung von Ganzkörper-Schwingungen auf den Menschen - Teil 1: Allgemeine Anforderungen (Mechanical vibration and shock - Evaluation of human exposure to whole-body vibration - Part 1: General requirements).

[6] Sahnaci, C. (2013): Menscheninduzierte Einwirkungen auf Tragwerke infolge der Lokomotionsformen Gehen und Rennen: Analyse und Modellierung. (Human-induced effects on structures as a result of locomotion forms of walking and running: Analysis and modelling.) Dissertation, Ruhr-Universität Bochum.

[7] Seiler, C. und S. Hüttner (2004): Ein einheitliches Modell zur Beschreibung von Fussgängerlasten für verschiedene Bewegungsarten-Theorie, experimentelle Messungen und praktische Anwendung (A unified model for describing pedestrian loads for different types of motion - theory, experimental measurements and practical application). In: Bauingenieur, 79: S. 483–496.

[8] VDI 2057 (2017): Blatt 1 - Einwirkung mechanischer Schwingungen auf den Menschen - Ganzkörper-Schwingungen (Human exposure to mechanical vibrations - Whole-body vibration).

[9] Ziegler, A. (2017): Schwingende Balkone (Vibrating balconies). In: Bauwerksdynamik und Erschütterungsmessungen, S. 167–177. Springer Fachmedien Wiesbaden. doi:10.1007/978-3-658-16054-8_7.